Data Provided:
Formula sheet and physical constants

DEPARTMENT OF PHYSICS & ASTRONOMY **Autumn Semester 2010-2011**

OPTICAL PROPERTIES OF SOLIDS **2 hours**

Answer THREE questions.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

NOTE

In this examination you may assume without proof that the relationship between the complex refractive index of a material $\tilde{n} = n + i\kappa$ and its complex relative dielectric constant $\tilde{\varepsilon}_r = \varepsilon_1 + i\varepsilon_2$ is given by

$$n = \frac{1}{\sqrt{2}}\left[\varepsilon_1 + \left(\varepsilon_1{}^2 + \varepsilon_2{}^2\right)^{1/2}\right]^{1/2}$$

$$\kappa = \frac{1}{\sqrt{2}}\left[-\varepsilon_1 + \left(\varepsilon_1{}^2 + \varepsilon_2{}^2\right)^{1/2}\right]^{1/2} .$$

1 (a) Explain what is meant by *dispersion* in wave motion in general and as applied specifically to optics. [1]

(b) (i) Explain the origin of *normal dispersion* in a transparent optical material such as glass in the visible spectral region.
(ii) Discuss whether the dispersion would be normal or anomalous in the visible spectral region in a coloured optical material such as ruby. [2]

(c) Explain what is meant by *birefringence*, and state, with reasons, which of the following materials would be expected to be birefringent:
i) glass
ii) Coca Cola
iii) glass which has been strained
iv) diamond (cubic structure)
v) sapphire (trigonal structure)
vi) sodium chloride (face centred cubic structure) [3]

(d) A quarter wave plate designed for use at 633 nm is made from quartz, which has ordinary and extraordinary refractive indices of 1.544 and 1.553 at this wavelength. Calculate the thickness of the plate, and explain how it can be used to convert linearly polarized light into circularly polarized light. [2]

(e) A Kerr cell contains a non-birefringent medium with electrodes applied, so that birefringence can be induced by applying an electric field. The induced birefringence is given by $\Delta n = \lambda K E^2$, where λ is the wavelength, K is the Kerr coefficient, and E is the electric field strength. A particular Kerr cell with a length of 2 cm and $K = 5\times10^{-14}$ mV^{-2} is placed between crossed polarizers. Calculate the field strength that would have to be applied to maximize the transmission through the system when the incoming light is linearly polarized parallel to the first polarizer. [2]

2 (a) Sketch the absorption spectrum that you would expect to observe near the band gap at room temperature for:

(i) a direct gap semiconductor with a small exciton binding energy compared to $k_B T$;

(ii) a direct gap semiconductor with a large exciton binding energy compared to $k_B T$. [2]

(b) Silicon is an indirect-gap semiconductor with a band gap of 1.12 eV at room temperature, while CdTe is a direct-gap semiconductor with a band gap of 1.474 eV at room temperature.

(i) How does the band edge absorption spectrum of silicon change as the temperature is decreased from room temperature to 4 K? [1]

(ii) Calculate the maximum current that can be generated in a silicon photodiode when illuminated by a laser beam of wavelength 850 nm and power 5 mW. [1]

(iii) Give one advantage and one disadvantage of using silicon as opposed to CdTe in solar cells. [2]

(iv) Use the data for CdTe given below to calculate the wavelength of the n = 1 exciton of CdTe at 4 K. [2]

(c) The absorption edge of a sample containing CdTe quantum dots is found to occur at 700 nm at room temperature.

(i) Explain what is meant by a *quantum dot*. [1]

(ii) On the assumption that the dots have a spherical shape, estimate the size of the dots. [1]

Data for CdTe:

Band gap at 4 K = 1.605 eV
Band gap at room temperature = 1.474 eV,
Electron effective mass $m_e^* = 0.099\ m_e$,
Hole effective mass $m_h^* = 0.3\ m_e$,
Relative dielectric constant $\varepsilon_r = 9.0$.

3 (a) Explain the meaning of the terms *photoluminescence*, *electroluminescence*, and *cathodoluminescence*. [1]

(b) Explain what is meant by the *quantum efficiency* of an electroluminescent light source. Why do semiconductors with indirect band gaps have very low electroluminescent quantum efficiencies? [2]

(c) In a photoluminescence experiment on bulk GaAs at 4 K, a laser injects electron hole pairs with a density of 1.0×10^{18} cm^{-3}. Relevant band data for GaAs at 4 K are given below.

(i) Calculate the Fermi energies of the electrons and holes. You may assume without proof that the density of states for a free particle of mass m is given by:

$$g(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} .$$ [1]

(ii) Sketch the emission spectrum that you would expect to observe, identifying clearly the energies of the upper and lower limits of the emission spectrum. [2]

(iii) How does the emission spectrum change as the carrier density is reduced? [1]

(d) Most laser diodes made nowadays incorporate quantum wells in the active region. Why is this? [1]

(e) A p-i-n diode contains GaAs quantum wells with a width 10 nm in the i-region.

(i) Estimate the emission wavelength that would be observed when the diode is operated at 4 K in forward bias. [1]

(ii) How would you expect the photoluminescence spectrum to vary with the voltage when the device is operated in reverse bias ? [1]

Data for GaAs at 4 K:

Band gap = 1.519 eV

Electron effective mass $m_e^* = 0.067\ m_e$,

Hole effective mass $m_h^* = 0.5\ m_e$,

4 (a) Show that the frequency dependence of the relative dielectric constant of a gas of free electrons is given by

$$\tilde{\varepsilon}_{\mathrm{r}}(\omega) = 1 - \frac{\omega_{\mathrm{p}}{}^2}{\omega^2 + i\gamma\omega} \ ,$$

where ω is the angular frequency, ω_{p} is the plasma frequency, and γ is the damping constant. Your answer should clearly state the relationship between the plasma frequency and *N*, the number of free electrons per unit volume. How is γ determined experimentally? [2]

(b) Use the formula for $\tilde{\varepsilon}_{\mathrm{r}}(\omega)$ above to explain why metals are expected to have near 100% reflectivity for frequencies below the plasma frequency. [1]

(c) Account qualitatively for the fact that the measured reflectivity of aluminium is about 90% in the visible spectral region, even though the plasma frequency corresponds to a wavelength in the vacuum ultraviolet spectral region. [1]

(d) Rubidium metal is found to be transparent for wavelengths shorter than 360 nm.
(i) Use this information to estimate the electron density in rubidium.
(ii) The actual electron density in rubidium is equal to 1.15×10^{28} m^{-3}. Compare this to the value calculated for rubidium in part (i), and account for any discrepancy. [2]

(e) The plasma frequency of copper lies in the ultraviolet spectral region, but copper metal, which has an electronic configuration of $3d^{10}4s^{1}$, is coloured. What causes this colouration? [1]

(f) Explain what is meant by a *bulk plasmon*, and discuss how such a plasmon might be observed experimentally in:
(i) a metal like aluminium,
(ii) a heavily-doped n-type semiconductor. [2]

(g) In an actual experiment on aluminium, two different types of plasmon are observed. Account for this observation, and state the relationship between the plasma frequencies of the two different types of plasmon. [1]

5 The frequency dependence of the relative dielectric constant of an ionic crystal in the infrared spectral region is given by:

$$\tilde{\varepsilon}_{\mathrm{r}}(\omega) = \varepsilon_{\infty} + (\varepsilon_{\mathrm{st}} - \varepsilon_{\infty})\frac{{\omega_0}^2}{({\omega_0}^2 - \omega^2 - i\gamma\omega)} \ ,$$

where ω is the angular frequency.

(a) (i) Explain the meaning of the symbols ε_∞, $\varepsilon_{\mathrm{st}}$, ω_0 and γ in this formula. [1]

(ii) Use the formula to explain why a lightly-damped ionic crystal reflects strongly in a range of infrared frequencies called the *Reststrahlen* band, stating the upper and lower angular frequency limits of this band. [2]

(b) The static and high frequency dielectric constants of InAs are 14.9 and 12.3 respectively. The transverse optic phonons of InAs have a frequency of 6.6×10^{12} Hz.

(i) What is the reflectivity at very low frequencies? [1]

(ii) What are the upper and lower wavelengths of the *Reststrahlen* band? [1]

(iii) The reflectivity is found to drop to near zero at a frequency just above the *Reststrahlen* band. Estimate the wavelength at which this occurs. [1.5]

(iv) Estimate the reflectivity at a wavelength of 43 μm, given that $\gamma = 2.5\times10^{11}\ \mathrm{s}^{-1}$. [2.5]

(v) Account for the large value of γ. [1]

END OF QUESTION PAPER