PHY202 – Quantum Mechanics
Tutorial sheet 1

Plane waves

1. It was shown in class that a free particle with mass m moving in the +ve x direction is described by a plane wave with momentum p and energy $E = p^2/2m$ given by $\Psi(x,t) = \Psi_0 \exp\left[i/\hbar\,(px - Et)\right] = \Psi_0 \exp\left(-iEt/\hbar\right) \exp\left(ipx/\hbar\right)$.

(a) By making a substitution $x \to -x$ show that a corresponding plane wave moving in the $-$ve x direction is given by $\Psi(x,t) = \Psi_0 \exp\left[-i/\hbar\,(px + Et)\right] = \Psi_0 \exp\left(-iEt/\hbar\right) \exp\left(-ipx/\hbar\right)$.
(b) Show that the plane wave function $\Psi(x,t) = \Psi_0 \exp\left[i/\hbar\,(px + Et)\right]$ with a 'wrong' sign of energy is unphysical. Justify your answer.

Infinite potential wells

2. In class we solved the time independent Schrödinger equation (TISE) for an infinite potential well, $V(x) = 0$ for $0 \le x \le L$ and $V(x) = \infty$ otherwise, to obtain the wavefunctions $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$ and the corresponding energies $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$, where $n = 1, 2, 3, \ldots$

Use the expression $\sin\alpha = (e^{i\alpha} - e^{-i\alpha})/2i$ to show that the wavefunctions $\psi_n(x)$ are a sum of two plane waves moving in the opposite directions. How do you interpret the resulting states?

3. (a) By considering the TISE, find the wavefunctions and the energies for the potential

$$V(x) = V_0, \qquad 0 \le x \le L, \qquad \text{and} \qquad V(x) = \infty, \quad \text{otherwise},$$

where V_0 is some (real) constant. Compare them to the case when $V_0 = 0$.
(b) By considering a general form of the TISE with potential energy $V(x)$, show that, generally, if $V(x)$ is changed everywhere by the constant V_0, the form of the time-independent wavefunctions is unchanged. What is the effect on the energy eigenvalues?

4. Consider an infinite potential well given by

$$V(x) = 0, \quad -\tfrac{L}{2} \le x \le \tfrac{L}{2}, \qquad \text{and} \qquad V(x) = \infty, \quad \text{otherwise},$$

which is the case considered in class but shifted to the left by $\frac{L}{2}$. Start from the wavefunctions $\psi_n(x)$ for the well between 0 and L and, by making the substitution $x \to x = x' + \frac{L}{2}$, where x' corresponds to the symmetric well, show that the solutions of the symmetric well are given by $\psi_n(x) = \sqrt{\frac{2}{L}} \cos \frac{n\pi}{L} x$ for n odd and $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$ for n even. (You don't need to explicitely solve the TISE to verify that they are the solutions for the shifted potential well. Remember to make a simple change of notation $x' \to x$ at the end.)

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