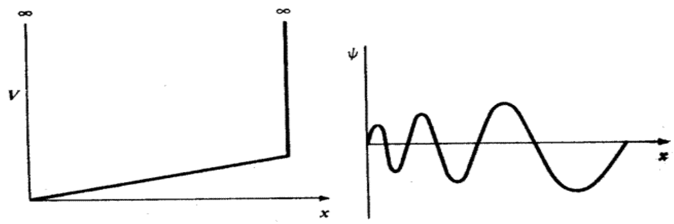
Quantum Mechanics - PHY202
Tutorial Questions 2

Sketching Wavefunctions

1. What is meant by *parity* in quantum mechanics? What properties must a potential have in order that the wavefunctions have definite parity?
   If wavefunctions have definite parity, why is does the ground state always have *even* parity?

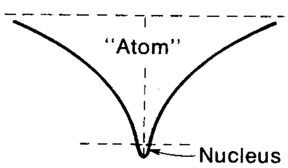
2. In *any* attractive potential the ground state wavefunction has no nodes. Why is this so?

3. The figure below shows a potential well and one of the possible wavefunctions for a particle in the well. Explain why the 'wavelength' of the wavefunction varies as it does.

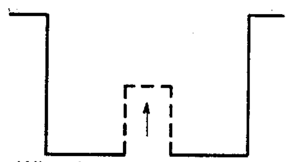


4. Sketch wave functions for the following potentials

(a)

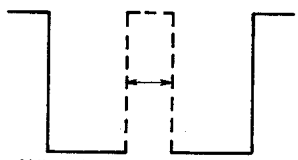


(b)



What happens as central barrier grows from zero to infinite *height*?

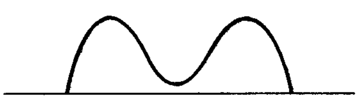
(c)



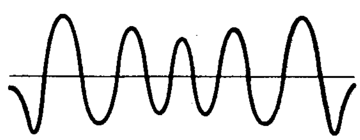
What happens as central barrier grows from zero to full *width*?

5. Sketch potentials which would give rise to the following wavefunctions

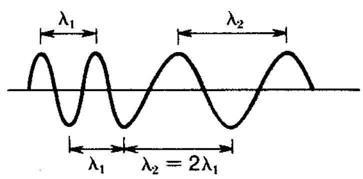
(a)



(b)



(c)



Operators and Measurement

6. Give examples of eigenvalue equations, stating eigenvalues and eigenfunctions, for
(a) the energy operator;
(b) the momentum operator;
(c) the parity operator.

7. Suppose that $\phi_1(x)$ is a plane (or travelling) wave, $A\exp(ikx)$.
(a) Show that this is an eigenstate of the momentum operator. What is its momentum?
(b) What is $\phi_2(x) = \hat{P}\phi_1(x)$? (where $\hat{P}$ is the parity operator).
(c) Using $\phi_1(x)$ and $\phi_2(x)$ construct states of definite parity.
(d) Show that these states of definite parity satisfy the parity eigenvalue equation.

8. A particle in an infinite square well, $V(x) = 0$ for $0 < x < L$, $V(x) = \infty$ otherwise, has the time independent wavefunction:

$$\psi(x) = A\left(2\sin\frac{\pi x}{L} + \sin\frac{2\pi x}{L}\right)$$

(a) By exploiting the orthonormality of the expansion functions, find the value of the normalization factor A.
(b) If a measurement of the energy is made, what are the possible results? What is the probability associated with each result?
(c) Using the results of (b) deduce the average energy and express it as a multiple of the energy E_1 of the lowest eigenstate.
(d) Calculate the *expectation value* of E using $\langle E \rangle = \int \psi^* \hat{H}\psi dx = \int \psi^* \left(-\frac{\hbar^2}{2m}\right)\frac{\partial^2\psi}{\partial x^2}dx$ and verify that this is identical to the result of (c).
(e) If the energy is measured, as in (b), what is the form of the wavefunction after the measurement?
(f) If the energy is immediately re-measured, what will be the probabilities of the possible outcomes now?

9. What does it mean to say that certain operators *commute*? Give examples of operators that commute and of operators that do not commute. (Hint, see also Q.2.)

10. Use the uncertainty principle to explain, in the simplest terms, why a zero point energy is observed for any particle in a bound state.

L. Roszkowski