Addition to Topic 10 on optical absorption.

Deduction of formula for the joint density of states $N(\hbar\omega)$, which is number of states per unit energy (and per space unit volume), associated with direct optical transition in semiconductors

For direct transition conservation of energy and momentum to be fulfilled:

$$\hbar\omega = E_g + \hbar^2 k_e^2 / 2m_e^* + \hbar^2 k_h^2 / 2m_h^* \qquad \text{Eq. 1)}$$

$$k_{ph} = k_e + k_h \qquad \text{Eq. 2)}$$

$$|k_e| \approx |k_h| \equiv k$$

$m^*_{e,h}$-effective electron (e) and hole (h) masses;
$k_{e,h}$ –electron and hole wavevectors
k_{ph}-photon wavevector

a) Number of states per volume element $d\vec{k} = dk_x dk_y dk_z$ in momentum space (and per unit volume in real space) $= 2d\vec{k}/(2\pi)^3$. Factor 2 is for spin.

b) In spherical coordinates $d\vec{k}/(2\pi)^3 = 2k^2 dk \sin\theta d\theta d\varphi /(2\pi)^3$

c) Number of states per interval dk in momentum space=

$$2k^2 dk \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta /(2\pi)^3 d\theta = \frac{8\pi k^2 dk}{(2\pi)^3}$$

d) Differentiating Eq.1) obtain $d(\hbar\omega) = \hbar^2 k dk (\frac{m_e^* + m_h^*}{m_e^* m_h^*})$;

From Eq.1) obtain $k = (\hbar\omega - E_g)^{1/2} \frac{\sqrt{2}}{\hbar} \left(\frac{m_e^* m_h^*}{m_e^* + m_h^*} \right)^{1/2}$.

Substituting **d)** to **c)**
obtain

$$\frac{8\pi k^2 dk}{(2\pi)^3} = \frac{8\pi(\hbar\omega - E_g)^{1/2} \left(\frac{2}{\hbar^2} \frac{m_e^* m_h^*}{m_e^* + m_h^*} \right)^{1/2} \frac{m_e^* m_h^*}{m_e^* + m_h^*} \frac{d(\hbar\omega)}{\hbar^2}}{(2\pi)^3} = \frac{(\hbar\omega - E_g)^{1/2} \left(\frac{2m_e^* m_h^*}{m_e^* + m_h^*} \right)^{3/2}}{2\pi^2 \hbar^3} d(\hbar\omega)$$

=> number of states per unit energy (and per space unit volume), which is joint density of states associated with the transition:

$$N(\hbar\omega) = \frac{\left(2 \frac{m_e^* m_h^*}{m_e^* + m_h^*} \right)^{3/2}}{2\pi^2 \hbar^3} (\hbar\omega - E_g)^{1/2}$$