Syllabus

1. Summary of key properties of semiconductors and motivation for low dimensional structures
2. Alloy semiconductors, lattice matched and mismatched structures
3. Growth techniques for quantum wells
4. Effect of 2, 1 and zero dimensional quantisation on properties of electrons and holes. Wavefunctions, density of states
5. Optical properties of wells, wires and dots. Absorption and emission spectra. What they tell us and what controls their properties
6. Coupled quantum wells and superlattices
7. Resonant tunnelling and the quantum cascade laser
8. Quantum confined Stark effect
9. Transport properties. Modulation doping
10. Growth techniques for quantum dots
11. Modern day physics and applications of quantum dots
Suitable textbooks:

1. J Hook and H Hall chapter 14 and pages 192-196
2. The Physics of Low Dimensional Semiconductors (J H Davies, Cambridge)
4. Electronic and Optical Properties of Semiconductor Structures (J Singh Cambridge)
5. Quantum Wells, Wires and Dots, (P Harrison, Wiley)
6. Low Dimensional Semiconductors, (M J Kelly, Oxford)
7. Quantum Semiconductor Structures (C Weisbuch and B Vinter Academic Press)
The influence of semiconductor devices is all-pervasive in life in the 21st century. The range of applications includes, to name just a few:

1. Integrated circuits in computers
2. High frequency components in mobile and satellite communications
3. Light emitting diodes for displays, car indicators and break-lights and lighting
4. Lasers in cds, dvds, blue-ray, laser pointers
5. Lasers for fibre optic communications
6. Any others??

Many of these vital parts of everyday life rely on semiconductor structures of reduced dimensionality.

The understanding of the basic physics underlying such structures, how they are made and their key properties forms the basis of this course.

We first begin with a summary of the basic properties of semiconductors required to underpin much of the course.
45nm transistors. Intel web site

Multi-quantum well

Vertical cavity and edge emitting lasers (Ledentsov 2002)

Bookham tunable telecommunications laser

Internet, optical data storage, lighting, displays, wireless and satellite communications
Key Properties of Semiconductors

1. They have a band gap separating valence and conduction bands. These lie in the range 0.2 to 3.5eV approximately. Typical examples include……

2. Direct and indirect band gaps

3. Direct gap semiconductors are important for optical applications, and some electronic applications

4. Silicon and germanium have indirect gaps. Silicon dominates electronics applications

5. Their conductivity can be controlled by doping

6. Examples of dopants

7. Their band gaps can be controlled by composition and by control of layer thickness

8. We will be mainly concerned in this course with III-V semiconductors – see periodic table
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Can vary band gap by controlling composition

The following graph plots band gap versus lattice constant for a variety of III-V semiconductors.

Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).
Multi Quantum Well Structure
By controlling compositions of different layers can create potential wells for electrons and holes. Reduced dimensionality results in size quantisation. This is the basis of low dimensional structures: control of size to give new physical properties.

Quantum well

Total difference in band gaps distributed in ratio of ~60:40 between conduction and valence bands.

For Al composition of 0.33
\[ \Delta E_c \sim 240 \text{meV} \]
\[ \Delta E_v \sim 160 \text{meV} \]
Transmission electron microscope cross-sections

GaAs-AlGaAs multi quantum well structure

InGaAs-InP multi quantum well structure

Need layer thicknesses to be order of de Broglie wavelength of electrons (≤30nm) to realise quantum effects
Lattice matched and lattice mismatched structures

Lattice matching: aim to have lattice constant of material A same as that for material B
Can vary band gap by controlling composition

The following graph plots band gap versus lattice constant for a variety of III-V semiconductors

Lattice Matched Growth:
Growth of material layers one on top of the other with same lattice constant, and with same lattice constant as substrate (see slide 7) e.g.

1. GaAs, AlAs and AlGaAs alloys
2. InGaAs alloy on InP with specific composition of InGaAs. 53% In composition
InGaAs alloy on InP with specific composition of InGaAs

GaAs, AlAs 0.3% difference in lattice constant
InAs, GaAs 7% difference

+GaN \~ 3.5eV
Lattice matched growth: schematic

Layers A, B have same lattice constant

Lattice matched growth
But **lattice mismatched growth** is also possible

At least one of layers is then strained – termed strained layer growth

**Restriction on thickness that can be grown** (‘critical thickness’)

In-plane lattice constant is same as that of substrate

- Material is compressed in plane
- Extended vertically

*FIGURE 3.12. Growth of In$_x$Ga$_{1-x}$As on a GaAs substrate. (a) Separate layers at equilibrium. (b) Thin layer of InGaAs on GaAs. The InGaAs is strained to conform to the lattice constant of GaAs in the plane of the heterojunction. (c) Thicker layer, where the strain has relaxed due to a misfit dislocation at the heterointerface, shown by an asterisk.*

Davies p97
Constraint:

- Strain energy builds up with thickness.
- Until strain energy becomes greater than energy to form dislocations in lattice.
- Dislocations highly undesirable.
- But mismatched growth gives high quality up to critical thickness.

Very great flexibility in band gaps and layer properties if structures designed correctly.

Combination of alloy composition, suitable lattice matching, controlled degree of mismatch and use of quantum wells gives control of emission wavelength used in modern day light emitters.
Crystal Growth Techniques for Low Dimensional Structures

Need control on 1-30nm scale (why?)

1. Molecular Beam Epitaxy (MBE)
2. Metal organic chemical vapour deposition (MOCVD) or Metal Organic Vapour Phase Epitaxy (MOVPE)
MBE Schematics
Molecular beam epitaxy growth

- Molecular beams
- Evaporation sources Knudsen cells

- Layer by layer growth (~1 µm/hour)
- Nanometre scale thickness control
- Surface analysis techniques (RHEED reflection high energy electron diffraction)
- Shutters on sources
- Sample rotation
- Ultra high vacuum ~10^{-10} Torr
- Heated substrate ~600°C
Molecular Beam Epitaxy V90 Production Geometry Reactor

See also http://www.shef.ac.uk/eee/nc35t/MBEepi.pdf
Metal organic chemical vapour deposition (MOCVD)

• Layer by layer growth
• Flowing gases over heated substrate
• Typical reaction

\[
\left( \text{CH}_3 \right)_3 \text{Ga} + \text{AsH}_3 \rightarrow \text{GaAs} + 3\text{CH}_4
\]

• Near atmospheric pressure
• Reaction takes place over substrate
• Somewhat less complex equipment than MBE
• Gases switched to change composition by mass flow controllers
• Growth rate (~1-5μm/hour)
• Nanometre scale thickness control
MOCVD schematics

- Ga(CH₃)₃
- Al(CH₃)₃
- Zn(C₂H₅)₂

For acceptors:
- Silica Reactor
- Waste gases

For donors:
- R.F. induction heating
- Substrate (~600°C)

- AsH₃/H₂
- H₂Se/H₂

Automated valves

KEY:
- Bellows valve
- Ball valve
- Vacuum throttle valve
- Liquid bubbler
- Mass flow controller
- Rotameter
- Gas cylinder
- Reactor barrel
- Susceptor
- Scrubber
- Burn box
- Particle filter
- MFC (Mass Flow Controller)
- Vacuum pump
- N₂
Metal-organic chemical vapour deposition reactor chamber

See also http://www.shef.ac.uk/eee/nc35t/MOVPEepi.pdf
Quantum wells, wires and dots

Quantum well – quantisation in one dimension
Quantum wire – quantisation in two dimensions
Quantum dot – quantisation in all three dimensions

Quantum wells require layer by layer growth (MBE or MOCVD growth as described above, with switching between layers)

Wires and dots need additional techniques

Will describe growth techniques for dots later
Quantum Wells

- Infinite well
- Finite well
- Density of states

Applications: LEDs, lasers, modulators
Infinite well: Wavefunctions zero at boundaries of well

Hook and Hall, p400
Infinite potential well (Schiff p38)

Schrodinger equation in one dimension

\[
\left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V \right) \psi = E \psi
\]

V=0 for \(-a < x < a\)

\[
\left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) \psi = E \psi
\]

Solution of form \(\psi = A \sin \alpha x + B \cos \alpha x\)

\[
-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = \frac{\hbar^2}{2m} \left( A \alpha^2 \sin \alpha x + B \alpha^2 \cos \alpha x \right) = E \left( A \sin \alpha x + B \cos \alpha x \right)
\]

\[
\therefore E = \frac{\alpha^2 \hbar^2}{2m}, \alpha = \left( \frac{2mE}{\hbar^2} \right)^{1/2}
\]
\[ V = \infty \text{ at } x = \pm a \text{ requires } \psi = 0 \text{ at } x = \pm a \]

\[ \therefore 0 = A \sin \alpha a + B \cos \alpha a \quad (1) \]
\[ 0 = -A \sin \alpha a + B \cos \alpha a \quad (2) \]

To satisfy (1) \( A = 0, \cos \alpha a = 0 \)
(2) \( B = 0, \sin \alpha a = 0 \)

For (1) \( \alpha a = n\pi/2 \quad n \text{ odd} \)
For (2) \( \alpha a = n\pi/2 \quad n \text{ even} \)

Thus \( \psi(x) = B \cos n\pi x/2a \quad n \text{ odd} \)
\[ \psi(x) = A \sin n\pi x/2a \quad n \text{ even} \]

\[ E = \frac{\hbar^2}{2m} \frac{n^2 \pi^2}{4a^2} = \frac{n^2 \hbar^2 \pi^2}{8ma^2} = \frac{n^2 \pi^2 \hbar^2}{2md^2} \quad \text{where } d = 2a \text{ is the width} \]
Alternative simple derivation for infinite well

Electron wavelength $\lambda = 2d/n$ (quantisation of electron wavelength)

Where $d$ is width of well

Wavevector $k = \frac{2\pi}{\lambda} = \frac{2\pi \pi}{2d} = \frac{\pi n}{d}$

Energy $E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2md^2}$

For 10nm well in GaAs ($m^* \sim 0.07m_e$), $E_1 = 45$meV
For finite well, wavefunctions leak into barrier
Effect on energy?

Quantum well

Kelly p252
Alternative simple derivation for infinite well

Electron wavelength $\lambda = \frac{2d}{n}$ (quantisation of electron wavelength)

Where $d$ is width of well

Wavevector

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\pi}{2d} = \frac{\pi n}{d}$$

Energy

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2md^2}$$

For 10nm well in GaAs ($m^* \sim 0.07m_e$), $E_1 = 45\text{meV}$

For finite well, $E_1 = 29\text{meV}$
Wavefunctions are quantised along $z$, but there is dispersion in the plane.

$$E_{tot} = E_n + \frac{\hbar^2}{2m_e} \left( k_x^2 + k_y^2 \right)$$

Quantised energy along $z$, the growth direction ($x$, $y$ are in plane).

Consequences for density of states, optical absorption.

One dimensional quantisation

\textit{FIGURE 4.9.} Quasi-two-dimensional system in a potential well of finite depth. Electrons with the same total energy can be bound in the well (A) or free (B).
Density of States (DOS) in Systems of Reduced Dimensionality

Important property which underlies new functions

First recall DOS in 3D

Free particle wavefunction satisfying periodic boundary conditions:

$$\psi_k(r) = e^{ik \cdot r}$$

$$k = 0, \pm \frac{2\pi}{L} \ldots \pm \frac{N\pi}{L}$$
In 3D, one allowed value of wavevector in volume element of $(2\pi/L)^3$

Thus up to energy $E$, wavevector, number of states

$$2 \left(\frac{2\pi}{L}\right)^3 \frac{4\pi k_F^3}{3} = \frac{V}{3\pi^2} k^3 = N \quad (1)$$

$N$ is number of particles, $V$ volume

But $E = \frac{\hbar^2 k^2}{2m}$

Substitute in (1) for $k$, $\rightarrow N = \frac{V}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{3/2}$

Density of states, number of states per unit energy range:

$$D(E) = \frac{dN}{dE} = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$$
Density of States in Two Dimensional System

In 2D, one allowed value of $k$ in area element $(2\pi/L)^2$

$$2\left(\frac{2\pi}{L}\right)^{-2} \pi k^2 = N$$

$$k = (2\pi n)^{1/2}$$

$$n = N / A$$

$$E = \frac{\hbar^2 \pi n}{m}$$

$$D(E) = \frac{dn}{dE} = \frac{m}{\hbar^2 \pi}$$

Constant independent of energy in 2D

The states which contribute are the in-plane states which are not quantised

(see slide 31)
Fig. 14.3  Density of states per unit area for free electrons in a thin film. $E_1$, $E_2$, ... are the bound state energies for the potential confining the electrons to the film (see Fig. 14.1(a)). The broken curve shows the energy dependence of the density of states for the three-dimensional electrons in a film of large thickness.
Transition Energies and Spectra

These are what are actually measured – not energy levels, density of states directly, but they give clear experimental evidence for above concepts.
Transition Energies

Both electron and hole states are quantised

Quantum well
Predicted absorption spectra (a and c) and densities of states (b)

Peaks in a and c are due to excitonic effects

Exciton: bound electron-hole pair, bound by mutual Coulomb interaction

As for donors, binding energy is given by

\[ E_x = \frac{\mu e^4}{2n^2 \hbar^2} \left( \varepsilon \varepsilon_0 \right)^2 \]

\[ E_x = 13.6 eV \left( \frac{\mu}{\varepsilon^2} \right) \]

But \( \mu \) is now the electron-hole reduced mass

Exciton binding energies are enhanced in GaAs quantum wells relative to bulk (from 4 to \(~10\)meV)
Excitons are stable at room temperature in quantum wells, in marked contrast to bulk semiconductors.

FIG. 1. Linear absorption spectra at room-temperature GaAs and MQW samples. (Note that the MQW absorption is underestimated because of thickness loss during etching.)

Key features?

Transition energies

\[ \hbar \omega = E_g + E_{el}(n) + E_h(n) - E_x \]

\( E_x \) is exciton binding energy (~10meV)
Absorption spectrum in two dimensional system (quantum well)

1. Resembles step function density of states
2. As well gets wider, absorption features get closer together in energy
3. Absorption strength from each sub-band the same
4. Exciton features appear due to Coulomb interaction between photo-excited electron and hole
5. Exciton effects are enhanced in 2D relative to 3D, since electron and hole are forced closer together by confinement
6. Exciton binding energy enhanced in quantum wells relative bulk. Hence excitons are stable at room temperature
7. Strongly allowed transitions are between electrons and holes with same sub-band index (since states have same parity and have large overlap. \( \Delta n=0 \) selection rule)
Allowed transitions in quantum wells

\[ \Delta n=0 \text{ selection rule} \]

Good approximation for symmetric wells

Transitions strong between states of the same parity and with large wavefunction overlap

Hook and Hall p404
Density of States in 3D, 2D, 1D, 0D

Evolution of DOS as dimensionality is reduced

DOS – density of states

f(E) Fermi-Dirac distribution function

n(E) injected carrier distribution as a function of energy
• Modification of density of states as dimensionality is reduced is highly beneficial since e.g. in laser devices as carriers injected, they are increasingly confined to be at the same energy. This leads to higher gain, lower threshold and more favourable laser properties.

• Furthermore quantisation leads to additional degree of control over wavelength

• What else gives control?
Applications of Quantum Wells

Main applications of quantum wells are as light emitting elements in light emitting diodes or lasers

Telecommunications wavelengths: InGaAs/AlGaInAs (1.3, 1.55μm)
Near infrared: (In)GaAs/AlGaAs (0.8-1.0 μm)
Red GaInP/AlGaInP (630-680nm)
Green InGaN/GaN (540-560nm)
Blue InGaN/GaN (380-440nm)

Design considerations include those of lattice matching, or controlled mismatch that we discussed earlier

Other applications:
Modulators, detectors
Coupled Quantum Wells and Superlattices

For real quantum wells with finite barriers, wavefunctions leak out of well, as we have seen earlier. Tunnelling of wavefunctions into barriers. (see also section on tunnelling)

If wells grown one on top of the other, with thin barriers then wavefunctions from adjacent wells overlap and coupled quantum wells (2 wells) and superlattices (many wells) can be formed
• $V_{12}$ is the coupling potential – determines how energy of electron in one well is modified by presence of other

• Without coupling, the energies in the two wells are the same

• Degeneracy is lifted by coupling, by amount $2V_{12}$

• Resultant wavefunctions have symmetric, antisymmetric symmetry

• Coupling arises due to leakage of wavefunctions into barrier and interaction

$$V_{12} = \int \psi_1 V \psi_2 dz$$
Coupling of many wells results in formation of superlattice

Similar to tight binding model for formation of energy bands in solids

Here coupling of say N wells, results in band with \((2^N)\) states

As barrier width \(L_B\) decreases, band width increases due to greater interaction between wells

What gives rise to dependence on well width

Weisbuch and Vinter p27
New three dimensional band structure created by coupling wells into superlattice

New bands termed minibands, separated by mini-gaps – see section on quantum cascade lasers

Davies p182
Resultant change in density of states

Application of superlattices in quantum cascade lasers. Efficient current transport and precise electron injection in complex multilayer structures

But growth is very challenging, since barrier widths typically of thickness 1nm to get large band widths and efficient vertical transport
Resonant Tunnelling

As opposed to classical particles, electron wavefunctions can penetrate through potential barriers due to the phenomenon of quantum mechanical tunnelling.

Can construct such tunnelling structures by controlled layer by layer growth.

Applications as mid infrared emitters and as high frequency oscillators.
Electron waves incident on barrier

Earlier example for coupled QWs and superlattices

Shows finite probability for transmission through barrier

Zero for classical particle for $E<V_0$

The single barrier tunnelling structure

Perfect transmission when barrier contains integral number of half wavelengths
Double barrier resonant tunnelling structure

1. Low voltage emitter energy less than $E$, $I=0$

2. Voltage ($V_1$) such that emitter aligned with $E_1$. Peak in current

3. $V>V_1$. $I \to 0$

4. $V=V_2$. Second current peak when emitter aligned with $E_2$
Further pictorial figure to assist in the understanding of previous page

FIGURE 5.13. Profile through a three-dimensional resonant-tunnelling diode. The bias increases from (a) to (d), giving rise to the $I(V)$ characteristic shown in (e). The shaded areas on the left and right are the Fermi seas of electrons.

Davies p174
Negative differential resistance regions give rise to application as high frequency oscillator up to 500GHz.

Major application as quantum cascade lasers
The Quantum Cascade Laser (QCL)

- A device based on 2D subbands (and quantum wells and barriers)
- As opposed to all other semiconductor lasers which rely on conduction to valence recombination, in a QCL transitions are only between confined conduction band states
- Wavelength is determined by width of layers, by design
- Composed of injector and active regions which are then repeated
- Structures have up to 500 layers, individual layers ~1nm thick. Each electron gives rise to up to 25 photons by emitting a photon in each period of the device
- Formation of superlattice minibands and electron tunnelling underlies operation

Diagram shows 2 periods of >25 period structure

Population inversion occurs between levels 3 and 2.

Transitions between conduction sub-bands: in mid infrared spectral region 4-20μm
Quantum cascade laser with superlattice injector region and ‘vertical’ lasing transition
Layer structure of QCL grown in Sheffield

Layer thicknesses are of order ~1nm to achieve effective tunnelling.

Resonant tunnelling is fundamental to device operation.

Lasers operate in mid infra-red and terahertz range, 4 to 50µm range.

Applications gas sensing, environmental monitoring.
The Quantum Confined Stark Effect

Quantum well in an electric field
Applied electric field gives rise to perturbation in quantum well Hamiltonian of

$$\Delta V = -eEz$$

$V$ is potential, $E$ electric field and $z$ is position

Leads to tilting of energy bands with two main results
(see next page)
Transition energy reduced by applied field

Selection rules changed
Now inversion asymmetric
Parity no longer good quantum number

Davies p259

Singh p429
Further application of quantum wells: exploitation of Quantum Confined Stark Effect as modulator in telecommunications systems

By applying modulation to Stark effect modulator impose signal on light transmitted into fibre 1.3 or 1.55µm
Inter-sub-band infra-red absorption

Spacing between first two sub-bands is ~150meV for a 4nm well (corresponds to wavelength of ~8μm)

Permits absorption in mid IR region in wide band gap material

New property not possessed by host semiconductor

Further application of quantum wells: as mid infrared detectors

In emission related structures: quantum cascade lasers
Note that selection rules are different for intra-band (conduction band to valence band) and inter-sub-band transitions

\( \Delta n = 0 \) and \( \Delta n = 1 \) in the two cases

Hint: the total electron wavefunction in a quantum well is:

\[
\psi = e^{ik\cdot r} \chi_n(z)u_k(r)
\]

\( \chi_n(z) \) is the slowly varying envelope function

\( u_k(r) \) is the Bloch function which labels the band (s-like conduction band, p-like valence band)

Electric dipole transition: operator \( \propto e\cdot r \), odd parity

For transition within conduction band, Bloch function does not change, and transition between envelope functions, which must thus have opposite parity. Thus \( \Delta n = 1 \).

For valence to conduction band transition (interband transition), transition is between Bloch functions which conserves parity. Envelope functions must thus have same parity and thus \( \Delta n = 0 \).
Valence to conduction band, interband absorption – electric dipole transition between Bloch functions, multiplied by overlap integral between envelope functions

Inter-sub-band absorption – electric dipole transition between envelope functions

Breakdown of selection rules in applied electric field – since parity no longer a good quantum number
Modulation Doping

• Creation of quantum wells for electrons by doping alone
• Charge transfer across heterostructure interface
• Suppression of electron scattering
• High frequency devices with important applications in mobile and satellite communications
Heterojunction between materials of two different compositions

But this is not an equilibrium situation

Electrons from donor dopants will transfer to narrower gap material

Two layers not in contact

Conduction bands aligned relative to vacuum level
Electrons flow from wider to narrower gap material until equilibrium is established, i.e. until chemical potentials on two sides are equalised.

For $\Delta E_c = 0$, $\Delta E_v = 0$ (conduction and valence band discontinuities zero between AlGaAs and GaAs) i.e GaAs homojunction (n, p regions both GaAs), we have

Band bending is due to space charge.
In case of modulation doped heterojunction, electrons transfer from wider gap (doped) material to narrower gap material. Leave positive space charge behind in AlGaAs. Negative space charge in GaAs.

From Poisson’s equation, this spatially varying charge distribution gives rise to bending of the bands.

\[ \frac{d^2V}{dx^2} = - \frac{\rho}{\varepsilon \varepsilon_0} \]

Quantum well for electrons formed at interface: one dimensional confinement.
Understanding band bending

• Positive space charge, $d^2V/dx^2$ is negative

• $d^2V/dx^2$ is positive on electron band diagram (since electron charge negative)

• i.e. upward curvature of bands (positive 2nd derivative) in AlGaAs region

• Electrons transfer to narrower gap region

• Negative space charge, $d^2V/dx^2$ is negative (on electron band diagram)

• Slope decreases with x on GaAs side

• Quantum well for electrons formed by triangular potential of electrostatic origin

• Transfer of charge occurs until $\mu_F$ on two sides equalised

• Triangular potential result of band bending and band discontinuity between two materials of differing band gap
• Importantly have achieved separation of electrons from dopant atoms – hence modulation doping

• Thus can suppress ionised impurity scattering of electrons, which in heavily doped semiconductors is the dominant scattering mechanism, particularly at low temperature

• Can minimise scattering by use of ‘spacer’ layers to further separate electrons from donors

• See next page for figures
Mobility versus temperature in bulk (3D) GaAs

Suppression of ionised impurity scattering in modulation doped structure

Ionised impurity scattering dominates in heavily doped material

Pfeiffer, Appl Phys Lett 55, 1888, 1989
For mobility of 12,000,000 cm²/V·sec

Electron mean free path is >50μm

‘Ballistic’ transport of electrons

Such structures used to observe the quantum and fractional quantum Hall effects, and as the basis for room temperature high frequency (2-10GHz) transistors

Technically high channel transconductance (high n and high μ) is the reason for excellent high frequency performance and important present day applications in mobile communications for example
Fermi energy of two dimensional electron gas

Typical electron density in channel may be $10^{16}$ m$^{-2}$

Density of states in two dimensions is $D(E) = m^* / \pi \hbar^2$

Thus for two dimensional electron gas of density $n_s$,

$$E_F = \frac{\hbar^2 \pi}{m^* n_s} = 34\text{meV}$$

for $n = 10^{16}$ m$^{-2}$
Semiconductor Quantum Dots

Fully confined systems in all three dimensions
Atom-like systems in the solid state
Will consider self assembled quantum dots
Applications in fundamental physics (access to single quantum states), and in telecommunications wavelength lasers to name just two
Density of States in 3D, 2D, 1D, 0D

Evolution of DOS as dimensionality is reduced

DOS – density of states

f(E) Fermi-Dirac distribution function

n(E) injected carrier distribution as a function of energy
Quantum dot growth

Characteristic of lattice mis-matched systems

Initially layer by layer

Islands form to relieve strain (lowers total energy of system)

So-called Stranski Krastanov techniques

Self assembly process
Self-Assembled Crystal Growth in Strained Systems (schematic) MBE Growth

(a) Stranski-Krastanow growth

(b) InAs-GaAs 7% lattice mismatch

(c) Note wetting layer

(d) Embedded in crystal matrix – like any other semiconductor laser or light emitting diode

InAs

GaAs
InAs Quantum Dots

M Hopkinson Sheffield

Dots form by self assembly – nucleation at sites on surface

There is a size and composition distribution of dots (result of self assembly process)
Higher growth rate, 0.3 monolayers/sec, density $\sim 5 \times 10^{10}$ cm$^{-2}$

Low growth rate 0.01 monolayers/sec), density $\sim 1 \times 10^9$ cm$^{-2}$
Quantum dot energy levels and transitions

Very strong confinement in z-direction

Atom-like energy levels from in-plane x, y confinement

s, p, d shells like atom (but degeneracies not exactly the same), since ??

Photoluminescence:
Electrons and holes excited at high energy
Relax in energy and fill dot states
In accord with requirements of Pauli principle
Photon emission – (photo)-luminescence
Photon absorption – optical absorption
Optical Spectra

Photoluminescence

Energy (eV)

Absorption

Excitation area ~100µm
Large numbers ~10^7 dots.
Linewidth ~30meV due to shape and size fluctuations

Transitions observed between n=1, 2 and 3 electron and hole levels
Δn=0 selection rule as for quantum wells
Spatially resolved emission

- Broadening due to space and size fluctuations removed by isolating individual dots

- Single dots optically isolated using apertures of ~500nm size.

- Emission spectrum breaks up into very sharp lines

- Linewidth (~1μeV) limited by radiative lifetime (Heisenberg uncertainty principle?)

- Ground (s-shell) and excited state (p-shell) emission observed

- Single dot applications up to ~50K
Filling of levels under optical illumination

- s-shells, 2 electrons in conduction band, 2 holes in valence band
- Exciton is electron-hole pair
- Thus if one electron hole pair created by external radiation, can accommodate both in respective s-shells
- Can add second e-h pair but with opposite spins.
- s-shell is then full
- Next carriers must then go in p-shell
- Next slide experimental evidence
Evidence for level filling in Quantum Dots

- Excitation power density ($P_{ex}$) controls average exciton occupancy ($N_X$)
  - $N_X << 1$ - single line
  - $N_X \sim 1-2$ - two groups of lines:
    - s-shell ($\sim 1345$ meV)
    - p-shell ($\sim 1380$ meV)

Evidence for
- Degeneracy of QD levels
- Forbidden transitions

*Physical Review B63, 161305R, 2001*
Applications of Quantum Dots

Will pick out two

Quantum dot lasers
Single photon sources
(also much fundamental physics of type carried out in Sheffield)
Quantum Dots for Telecommunications Lasers

Vertical cavity

Edge emitting

- Very low thresholds
- Very small temperature dependence of threshold current
- 1.3μm lasing on GaAs substrates

Semiconductor Laser Performance Versus Year

1.3μm quantum dot lasers for telecoms applications (InGaAs-based quantum dots)

Comparison of QD and quantum well lasers: temperature dependence of threshold current (DJ Mowbray lecture notes)

- Very low thresholds
- Very small temperature dependence of threshold current
- $1.3 \mu m$ lasing on GaAs substrates

$J_{th}$ nearly independent of temperature up to 300K – characteristic of 0D density of states
Single photon sources for quantum cryptography applications

Single photon source provides secure basis for quantum communication
Resistant to eaves-dropping
Single quantum dots well-suited to fabrication of single photon sources

*Emission spectrum of a single quantum dot for continuous excitation. 1X is the emission energy when an exciton recombines in an otherwise empty dot. 2X is the emission energy when an exciton recombines in a dot initially occupied by two excitons.*

Acknowledgement DJ Mowbray

Shifts in emission energy due to Coulomb interactions each time an exciton is added to a dot
If QD excited by pulsed laser, for each laser pulse, only one photon will be detected at the single exciton energy.

Hence single photon source

Second order correlation function

Two detectors

First detector detects first photon

Time delay till second detector detects a photon is recorded

Very small peak at time ‘t=0’ shows very good single photon source

Time at which photon is detected on first detector triggers counting electronics
Summary of applications of low dimensional structures

Quantum Wells (extended from slide 45)

Telecommunications wavelengths: InGaAs/AlGaInAs (1.3, 1.55μm)
Near infrared: (In)GaAs/AlGaAs (0.8-1.0 μm)
Red GaInP/AlGaInP (630-680nm)
Green InGaN/GaN (540-560nm)
Blue InGaN/GaN (380-440nm)
Mid infra-red quantum cascade lasers (3-50μm)
Modulators, detectors
High electron mobility transistors (GHz range)

Quantum Dots

Telecommunications wavelength sources
Quantum optics, cryptography applications